

A ŠMIGOC GLUING APPROACH FOR CONTROLLING JORDAN STRUCTURES

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Abstract. A list $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ of complex numbers is said to be realizable if it is the spectrum of an entrywise nonnegative A , which is called a realizing matrix for Λ . Λ is said to be universally realizable if there exists a nonnegative matrix with spectrum Λ for each Jordan structure allowed by Λ . The problems of realizability and universal realizability of a list of complex numbers are solved only for $n \leq 4$, which illustrates their difficulty.

In this work, the universal realizability problem with Jordan control is studied. In particular, a Šmigoc gluing approach for controlling the Jordan structures and constructing the corresponding nonnegative matrices associated to Λ is proposed. The idea is first to construct smaller auxiliary nonnegative matrices realizing the desired Jordan components with relatively small Perron roots, and then these auxiliary blocks are inserted into a real base matrix by means of Šmigoc gluing, preserving nonnegativity, the spectrum and the desired Jordan structure.

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