

MULTIPLICATIVELY PSEUDO SPECTRUM PRESERVING MAPS ON BANACH SPACES

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Abstract. Let $\mathcal{B}(X)$ be the algebra of all bounded linear operators on a complex Banach space X with $\dim X \geq 3$, and $\mathcal{A}, \mathcal{B}$ two subsets of $\mathcal{B}(X)$ containing all operators of rank at most one. For $\varepsilon \in (0, +\infty)$ and $A \in \mathcal{B}(X)$, let $\sigma_\varepsilon(A)$ and $r_\varepsilon(A)$ denote the ε -pseudo spectrum and the ε -pseudo spectral radius of A , respectively. We prove that a pair of surjective maps $\phi, \psi: \mathcal{A} \rightarrow \mathcal{B}$ satisfy

$$\sigma_\varepsilon(\phi(A)\psi(B)) = \sigma_\varepsilon(AB) \quad (A, B \in \mathcal{A})$$

if and only if there exist two bounded invertible linear operators U, V on X with $\|Vx\| = \|x\|$ for each $x \in X$, such that for all $A \in \mathcal{A}$, $\phi(A) = \mu V A U^{-1}$ and $\psi(A) = \nu U A V^{-1}$ for some μ and ν with $\mu\nu = 1$. We also characterize all surjective maps $\phi, \psi: \mathcal{A} \rightarrow \mathcal{B}$ satisfying

$$r_\varepsilon(\phi(A)\psi(B)) = r_\varepsilon(AB) \quad (A, B \in \mathcal{A}).$$

Furthermore, some known results are obtained as immediate consequences of our main results.

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