

ON ITERATIVE APPROXIMATE SCHAUDER FRAMES IN BANACH SPACES

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Abstract. Let X be a separable Banach space with the dual space X^* . The purpose of the paper is to investigate an approximate Schauder frame $(x_k, f_k)_{k \in \mathbb{Z}} \subset X \times X^*$ of the form $(W^k x_0, T^k f_0)_{k \in \mathbb{Z}}$ where $W \in B(X)$ and $T \in B(X^*)$ are invertible, namely, iterative approximate Schauder frames. A main result determines the operator W in terms of the adjoint of T and the frame operator. As a consequence, we also obtain some relationships between the norms of the operators W and T . Moreover, using the theory of q -Riesz basis ($1 < q < \infty$), we give sufficient conditions for an approximate Schauder frame to be iterative. Finally, we obtain some results on the duality construction of an iterative approximate Schauder frame; in particular, if $(W^k x_0, T^k f_0)_{k \in \mathbb{Z}}$ is shrinking, then its canonical dual also has an iterative structure.

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