

A SHORT PROOF OF FAGUNDES'S THEOREM

LAN LU AND YU WANG*

Abstract. In 2019, Fagundes used a tedious argument to prove that every multilinear polynomial on strictly upper triangular matrix algebras over any field is a vector space. The goal of the paper is to give a short proof of the result.

Mathematics subject classification (2020): 16S50, 15A54.

Keywords and phrases: Multilinear polynomial, strictly upper triangular matrix algebra, field.

REFERENCES

- [1] Q. CHEN, Y. L. LUO, Y. WANG, *Fagundes-Mello conjecture over a finite field*, Finite Fields Appl. **2025**, 105, 102620.
- [2] Q. CHEN, Y. WANG, *Images of generalized multilinear polynomials on upper triangular matrix algebras*, Axioms **2025**, 14 (9), 671.
- [3] P. S. FAGUNDES, *The images of multilinear polynomials on strictly upper triangular matrices*, Linear Algebra Appl. **2019**, 563, 287–301.
- [4] P. S. FAGUNDES, P. KOSHLUKOV, *Images of multilinear graded polynomials on upper triangular matrix algebras*, Canadian J. Math. **2023**, 75, 1540–1565.
- [5] P. S. FAGUNDES, T. DE MELLO, *Images of multilinear polynomials of degree up to four on upper triangular matrices*, Oper. Matrices **2019**, 13, 283–292.
- [6] I. G. GARGATE, T. C. DE MELLO, *Images of multilinear polynomials on $n \times n$ upper triangular matrices over infinite field*, Israel J. Math. **2022**, 252, 337–354.
- [7] N. JACOBSON, *Basic Algebra I*, 2nd ed., W. H. Freeman and Company: New York, USA, 1985.
- [8] Y. Y. LUO, Y. WANG, *On Fagundes-Mello conjecture*, J. Algebra **2022**, 592, 118–152.
- [9] T. DE MELLO, *The image of multilinear polynomials evaluated on 3×3 upper triangular matrices*, Comm. Math. **2021**, 29, 183–186.
- [10] Y. WANG, *The images of multilinear polynomials on 2×2 upper triangular matrix algebras*, Linear Multilinear Algebra **2019**, 67, 2366–2372, correction in Linear Multilinear Algebra **2019**, 67, i–vi.