

PROPER SPLITTINGS OF HERMITIAN OPERATORS

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Abstract. In this article we deepen the study of proper splittings of Hilbert space operators, with special emphasis on proper splittings of Hermitian operators. We improve characterizations given in [Fongi & Gonzalez, J. Math. Anal. Appl. **545** (2025) 129093] of the convergence of both the polar proper and the Moore Penrose proper splittings. On the other hand, we introduce new proper splittings and we compare their convergence with those of the polar and the Moore Penrose proper splittings.

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