

ON A RELATIONSHIP BETWEEN ORTHOGONAL PROJECTIONS AND TOEPLITZ OPERATORS ON POLY-BERGMAN SPACES OF THE UPPER HALF-PLANE: VERTICAL SYMBOLS

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Abstract. In the context of studying C^* -algebras generated by Toeplitz operators acting on the poly-Bergman space $\mathcal{A}_n^2(\Pi)$ of the upper half-plane Π , we introduce a system of all-but-one orthogonal projections in generic position. We show that the C^* -algebra generated by these orthoprojections is closely related to the C^* -algebra generated by all Toeplitz operators with vertical symbols satisfying boundary conditions. This result suggests a new approach in the study of Toeplitz operators acting on other reproducing kernel Hilbert spaces. Furthermore, the range of one of the orthoprojections herein has a reproducing kernel expressed in terms of the digamma and the Nielsen's beta functions. The harmonic function also emerges in this development.

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