

ANALYTIC α -MORREY SPACES AND VOLTERRA INTEGRAL OPERATORS

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Abstract. A family of analytic α -Morrey spaces $\mathcal{L}^{2,\lambda,\alpha}$ in the unit disc is introduced. Subsequently, some basic properties of $\mathcal{L}^{2,\lambda,\alpha}$ are established. Moreover, the boundedness, compactness, norm, and essential norm of the Volterra integral operators T_g and S_g acting on analytic α -Morrey spaces $\mathcal{L}^{2,\lambda,\alpha}$ are investigated comprehensively.

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