

GRAPHS WITH DISTINCT RECIPROCAL DISTANCE LAPLACIAN EIGENVALUES

SHIB SANKAR SAHA AND SWARUP KUMAR PANDA*

Abstract. In this article, we consider the following problem: *Characterize connected graphs G such that G has $k(\in \mathbb{N})$ distinct reciprocal distance Laplacian eigenvalues.* For $k = 2$, we provide the complete answer. For $k = 3$, graphs with three distinct eigenvalues are identified from a class of connected graphs. For $k = 4$, we prove that $K_{n,n} - e$ has four distinct reciprocal distance Laplacian eigenvalues for $n \geq 2$, where $K_{n,n}$ is complete bipartite and e is an edge in $K_{n,n}$. We also establish a relation between the reciprocal distance Laplacian spectrum and the Laplacian spectrum of a connected graph.

Mathematics subject classification (2020): 05C12, 05C50.

Keywords and phrases: Distance matrix, reciprocal distance matrix, Laplacian matrix, Laplacian eigenvalues, reciprocal distance Laplacian matrix, reciprocal distance Laplacian eigenvalues.

REFERENCES

- [1] M. AOUCHICHE, P. HANSEN, *Two Laplacians for the distance matrix of a graph*, Linear Algebra and its Applications **439** (2013), pp. 21–33, <https://doi.org/10.1016/j.laa.2013.02.030>.
- [2] M. AOUCHICHE, T. D. MORLEY, *Distance spectra of graphs: A survey*, Linear Algebra and its Applications **458** (2014), pp. 301–386, <https://doi.org/10.1016/j.laa.2014.06.010>.
- [3] F. AYOUBI, G. R. OMIDI, B. TAYFEH-REZAIE, *A note on graphs whose signless Laplacian has three distinct eigenvalues*, Linear and Multilinear Algebra **59** (6) (2011), pp. 701–706, <https://doi.org/10.1080/03081087.2010.489900>.
- [4] R. B. BAPAT, S. PANDA, *The spectral radius of the reciprocal distance Laplacian matrix of a graph*, Bulletin of the Iranian Mathematical Society **44** (4) (2018), pp. 1–6, <https://doi.org/10.1007/s41980-018-0084-z>.
- [5] R. O. BRAGA, R. R. DEL-VECCHIO, V. M. RODRIGUES, V. TREVISAN, *Trees with 4 or 5 distinct normalized Laplacian eigenvalues*, Linear Algebra and its Applications **471** (2015), pp. 615–635, <https://doi.org/10.1016/j.laa.2015.01.018>.
- [6] A. E. BROUWER, W. H. HAEMERS, *Spectra of Graphs*, New York: Springer, 2012.
- [7] F. R. K. CHUNG, *Spectral Graph Theory*, American Mathematical Society, Providence, RI, 1997.
- [8] D. CORNEIL, H. LERCHS, L. BURLINGHAM, *Complement reducible graphs*, Discrete Applied Mathematics **3** (1981), pp. 163–174, [https://doi.org/10.1016/0166-218X\(81\)90013-5](https://doi.org/10.1016/0166-218X(81)90013-5).
- [9] C. M. DA SILVA JR., M. A. A. DE FREITAS, R. R. DEL-VECCHIO, *A note on a conjecture for the distance Laplacian matrix*, Electronic Journal of Linear Algebra **31** (2016), pp. 60–68, <https://doi.org/10.13001/1081-3810.3002>.
- [10] M. DOOB, *Graphs with a small number of distinct eigenvalues*, Annals of the New York Academy of Sciences **175** (1970), pp. 104–110, <https://doi.org/10.1111/j.1749-6632.1970.tb56460.x>.
- [11] W. H. HAEMERS, G. R. OMIDI, *Universal adjacency matrices with two eigenvalues*, Linear Algebra and its Applications **435** (2011), pp. 2520–2529, <https://doi.org/10.1016/j.laa.2011.02.023>.
- [12] R. A. HORN, C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, 1990.
- [13] X. HUANG, Q. HUANG, *On graphs with three or four distinct normalized Laplacian eigenvalues*, Algebra Colloquium **26** (2019), pp. 65–82, <https://doi.org/10.1142/S1005386719000075>.

- [14] O. IVANCIUC, T.-S. BALABAN, A. T. BALABAN, *Design of topological indices. Part 4. Reciprocal distance matrix, related local vertex invariants and topological indices*, Journal of Mathematical Chemistry **12** (1993), pp. 309–318, <https://doi.org/10.1007/BF01164642>.
- [15] L. MEDINA, M. TRIGO, *Bounds on the reciprocal distance energy and reciprocal distance Laplacian energies of a graph*, Linear and Multilinear Algebra, (2020), pp. 386–412, <https://doi.org/10.1080/03081087.2020.1825607>.
- [16] L. MEDINA, M. TRIGO, *Upper bounds and lower bounds for the spectral radius of reciprocal distance, reciprocal distance Laplacian and reciprocal distance signless Laplacian matrices*, Linear Algebra and its Applications **609** (2021), pp. 386–412, <https://doi.org/10.1016/j.laa.2020.09.024>.
- [17] A. MOHAMMADIAN, B. TAYFEH-REZAIE, *Graphs with four distinct Laplacian eigenvalues*, Journal of Algebraic Combinatorics **34** (2011), pp. 671–682, <https://doi.org/10.1007/s10801-011-0287-3>.
- [18] B. MOHAR, *The Laplacian Spectrum of Graphs*, in: Y. Alavi, G. Chartrand, O. R. Oellermann, A. J. Schwenk (eds.), Graph Theory, Combinatorics, and Applications, vol. 2, Wiley, 1991, pp. 871–898.
- [19] P. ROWLINSON, *On graphs with just three distinct eigenvalues*, Linear Algebra and its Applications **507** (2016), pp. 462–473, <https://doi.org/10.1016/j.laa.2016.06.031>.
- [20] E. R. VAN DAM, *Regular graphs with four eigenvalues*, Linear Algebra and its Applications **226–228** (1995), pp. 139–162, [https://doi.org/10.1016/0024-3795\(94\)00346-F](https://doi.org/10.1016/0024-3795(94)00346-F).
- [21] E. R. VAN DAM, *Nonregular graphs with three eigenvalues*, Journal of Combinatorial Theory, Series B, **73** (1998), pp. 101–118, <https://doi.org/10.1006/jctb.1998.1815>.
- [22] E. R. VAN DAM, W. H. HAEMERS, *Which graphs are determined by their spectrum?*, Linear Algebra and its Applications **373** (2003), pp. 241–272, [https://doi.org/10.1016/S0024-3795\(03\)00483-X](https://doi.org/10.1016/S0024-3795(03)00483-X).
- [23] E. R. VAN DAM, G. R. OMIDI, *Graphs whose normalized Laplacian has three eigenvalues*, Linear Algebra and its Applications **435** (2011), pp. 2560–2569, <https://doi.org/10.1016/j.laa.2011.02.005>.