

SOME GENERALIZED REFINEMENTS AND REVERSES OF CONVEX FUNCTIONS AND APPLICATIONS

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Abstract. In this paper, we establish several new inequalities associated with different classes of convexity, including convex, log-convex, and p -convex functions, together with their refinements and reverses. As applications, we establish several refinements and reverses of Hölder-type inequalities involving the quadratic weighted geometric mean.

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