

REFINEMENTS OF THE HERMITE–HADAMARD INEQUALITY BY INCREASING FUNCTIONS AND THEIR PSEUDO–INVERSES

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Abstract. In this paper, we establish some generalizations and extensions of a recent refinement of the Hermite–Hadamard inequality by using the notion of pseudo-inverse. The relationship between the obtained refinements are studied. We generate sequences of refinements and analyze their convergence. Finally, some applications are given.

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