

ESTIMATES FOR THE NORM OF THE VOLTERRA OPERATOR IN WEIGHTED LEBESGUE SPACES

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Abstract. In this paper, we study the Volterra integral operator with so-called Oinarov's kernel acting between weighted Lebesgue spaces $L_{p,v}$ and $L_{q,u}$, where $1 < p, q < \infty$. We derive new two-sided estimates for the operator norm and provide refined upper and lower estimates explicitly expressed in terms of the kernel and the associated weights.

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