

REFINEMENTS OF SOME CLASSICAL INEQUALITIES VIA SUPERTERZATIC FUNCTIONS

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Abstract. Using superterzaticity of power functions we obtain new refinements of Hölder's, Minkowski's and Beckenbach-Dresher's inequalities with "turning point" $p = 3$. Some new two-sided versions of these inequalities for the intervals $1 < p \leq 2$ and $2 \leq p \leq 3$ are also stated.

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