

MORE GENERALIZATIONS ON SINGULAR VALUE AND NORM INEQUALITIES OF MATRICES INVOLVING SUBMULTIPLICATIVE FUNCTIONS

XIAOHUI FU*, PEILIN LIU AND CHUCHU TANG

Abstract. In this note, we obtain several singular value and unitarily invariant norm inequalities which generalize Al-Natoor et al.'s results (2025) in [1] and inspired by statements due to Audenh and Kittaneh (2012) in [2], we present the equivalent relations of some known inequalities involving nonnegative and increasing submultiplicative functions.

Mathematics subject classification (2020): 15A42, 15A18, 15A60, 47A30, 47B15.

Keywords and phrases: Singular value inequality, norm inequality, submultiplicative function.

REFERENCES

- [1] A. AL-NATOOR, S. K. KAUSHIK, F. KITTANEH, A. KUMAR, *Further singular value and norm inequalities for matrices*, Positivity. **29** (2025), <https://doi.org/10.1007/s11117-025-01126-y>.
- [2] W. AUDEH, F. KITTANEH, *Singular value inequalities for compact operators*, Linear Algebra Appl. **437** (2012) 2516–2522.
- [3] R. BHATIA, F. KITTANEH, *On the singular values of a product of operators*, SIAM J. Matrix Anal. Appl. **11** (1990) 272–277.
- [4] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York, 1997.
- [5] R. BHATIA, F. KITTANEH, *The matrix arithmetic-geometric mean inequality revisited*, Linear Algebra Appl. **428** (2008) 2177–2191.
- [6] J.-C. BOURIN, *A matrix subadditivity inequality for symmetric norms*, Proc. Amer. Math. Soc. **138** (2009) 495–504.
- [7] O. HRZALLAH, *Commutator inequalities for Hilbert space operators*, Linear Algebra Appl. **431** (2009) 1571–1578.
- [8] R. A. HORN, C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge University Press, 1991.
- [9] F. KITTANEH, *Norm inequalities for sums of positive semidefinite operators, II*, Positivity. **10** (2006) 251–260.
- [10] F. KITTANEH, *Inequalities for commutators of positive operators*, J. Funct. Anal. **250** (2007) 132–143.
- [11] F. KITTANEH, *Norm inequalities for commutators of positive operators and applications*, Math. Z. **258** (2008) 845–849.
- [12] F. KITTANEH, *Singular value inequalities for commutators of Hilbert space operators*, Linear Algebra Appl. **430** (2009) 2362–2367.
- [13] Y. TAO, *More results on singular value inequalities of matrices*, Linear Algebra Appl. **416** (2006) 724–729.
- [14] X. ZHAN, *Singular values of differences of positive semidefinite matrices*, SIAM J. Matrix. Anal. Appl. **22** (2001) 819–823.
- [15] X. ZHAN, *Matrix Theory*, Beijing: Higher Education Press, 2008, (in Chinese).