

ON SOME GENERALIZED INTEGRATION OPERATORS FROM BESOV SPACES TO BLOCH TENT SPACES

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Abstract. Let $H(\mathbb{D})$ be the class of all analytic functions on the open unit disk $\mathbb{D}$, $\vec{u} = (u_0, u_1, \dots, u_{n-1})$ with $u_l \in H(\mathbb{D})$ for $l = \overline{0, n-1}$. We study the following operators

$$T_{\vec{u}}^{(n)}(f)(z) = I^n(u_0^{(n)}f + u_1^{(n-1)}f' + \dots + u_{n-1}'f^{(n-1)})(z), \quad f \in H(\mathbb{D}),$$

where I is the integration operator

$$I(f)(z) = \int_0^z f(\zeta) d\zeta,$$

and I^n is the n th iteration of I , characterizing the boundedness and compactness of these operators acting from the Besov spaces B_p to the Bloch tent spaces for $p \geq 1$, and estimate the essential norm of the operators for $p > 1$.

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