

FURTHER INEQUALITIES FOR $(\mathcal{M}_\phi, G)$ h -CONVEX FUNCTIONS WITH APPLICATIONS

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Abstract. In this paper, we introduce several new properties and inequalities for $(\mathcal{M}_\phi, G)$ h -convex functions, complementing the existing results in this field. Additionally, we explore multi-term multiplicative refinements of the classical Jensen-type inequality for $(\mathcal{M}_\phi, G)$ h -convex functions. As an application, by selecting specific $(\mathcal{M}_\phi, G)$ h -convex functions, we present novel inequalities for unitarily invariant norms. Our results extend and generalize some recent publications in this field, including [5, 11, 12].

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