

REFINED UPPER BOUNDS FOR THE NUMERICAL RADIUS OF OPERATORS IN HILBERT SPACES

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Abstract. In this work, we employ some existing refinements of the well-known Cauchy–Schwarz and Buzano inequalities to derive improved upper bounds for the numerical radius of operator matrices on Hilbert spaces. As consequences, we obtain new upper bounds for the numerical radius of sums and differences of Hilbert space operators.

Mathematics subject classification (2020): 47A12, 47A30, 47A63, 47B33.

Keywords and phrases: Numerical radius, usual operator norm.

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