

BOUNDS FOR THE EUCLIDEAN $(A-M)$ -NUMERICAL RADIUS OF A PAIR OF OPERATORS IN HILBERT SPACES WITH APPLICATIONS

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Abstract. We define the p -arithmetic mean $(A-M)$ -numerical radius for the pair of operators $(A, B) \in \mathcal{B}(H)$ as

$$w_{p,v}(A, B) := \sup_{\|x\|=1} ((1-v)|\langle Ax, x \rangle|^p + v|\langle Bx, x \rangle|^p)^{1/p},$$

where $v \in [0, 1]$ and $p \geq 1$. For $p = 2$ we put $w_{e,v}(A, B) := w_{2,v}(A, B)$ and call it the *Euclidean $(A-M)$ -numerical radius*.

In this paper we obtain among others that, if $\alpha, v \in [0, 1]$, then the following upper bound

$$\begin{aligned} w_{e,v}^2(A, B) &\leq \max\{v, 1-v\} w^2(A-B) + \left\| (1-v)|A|^{2\alpha} + v|B|^{2\alpha} \right\| \\ &\quad \times \left\| (1-v)|A^*|^{2(1-\alpha)} + v|B^*|^{2(1-\alpha)} \right\|. \end{aligned}$$

and lower bound

$$\begin{aligned} w_{e,v}^2(A, B) &\geq \max \left\{ w \left[(1-v)^2 A^2 + v^2 B^2 \right], (1-v)vw(AB+BA) \right\} \\ &\quad + \max \{ (1-v)w(A), vw(B) \} \\ &\quad \times |w[(1-v)A + vB] - w[(1-v)A - vB]| \end{aligned}$$

are valid.

Applications for the numerical radius and Davis-Wielandt radius of an operator are also provided. Some integral inequalities of Hermite-Hadamard type are given as well.

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