

EXPLORING TORAL $[m, \mathcal{C}]$ -ISOMETRIES IN THE CONTEXT OF SEVERAL OPERATORS THEORY

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Abstract. The family of commuting m -isometric tuples has attracted significant attention in operator theory. In this work, we propose and analyze a new class, termed toral $[m, \mathcal{C}]$ -isometric tuples of commuting operators, which extends and unifies several existing frameworks. We develop a comprehensive structural theory for this class, including fundamental properties, and stability under natural operator-theoretic constructions.

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