

A NEW HARDY–HILBERT–TYPE INTEGRAL INEQUALITY INVOLVING ONE EXTENDED DERIVATIVE FUNCTION OF HIGHER-ORDER

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Abstract. By means of the weight functions, the idea of introducing parameters and the techniques of real analysis, a new Hardy–Hilbert-type integral inequality with two internal variables involving one extended derivative functions of m -order ($m \in \mathbf{N}_0$) is obtained. The equivalent statements of the best possible constant factor related to some parameters are considered. The equivalent forms, the operator expressions and the cases of reverses are provided.

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