

GENERALIZED UNCERTAINTY PRINCIPLE IN THE CONTEXT OF DIFFERENTIAL FORMS

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Abstract. This paper extends the uncertainty principle from function spaces to the more general space of differential forms. A brief introduction to differential forms provides a natural foundation for this generalization. By applying a general extension lemma, the Fourier transform is extended to differential forms, establishing the corresponding inversion theorem and Parseval's identity. Utilizing the method of phase derivatives and Cauchy's inequality, a new inequality is derived, leading to a generalized uncertainty principle within the space of differential forms.

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