

WEIGHTED HÖLDER WITH NUMERICAL RADIUS INEQUALITIES

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Abstract. The main aim of the current work is a discussion of the matrix Hölder inequality for unitarily invariant norms, with some applications to norm and numerical radius bounds.

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